



# GRAPHREACH: Position-Aware Graph Neural Network using Reachability Estimations



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## Objectives

An *inductive* GNN model that outputs node embeddings that are:

- **Holistic**: Maximum absorption of graph knowledge
- **Meaningful**: Real-world semantics are attended to
- **Robust**: Small alteration to graph does not affect embeddings

## Motivation

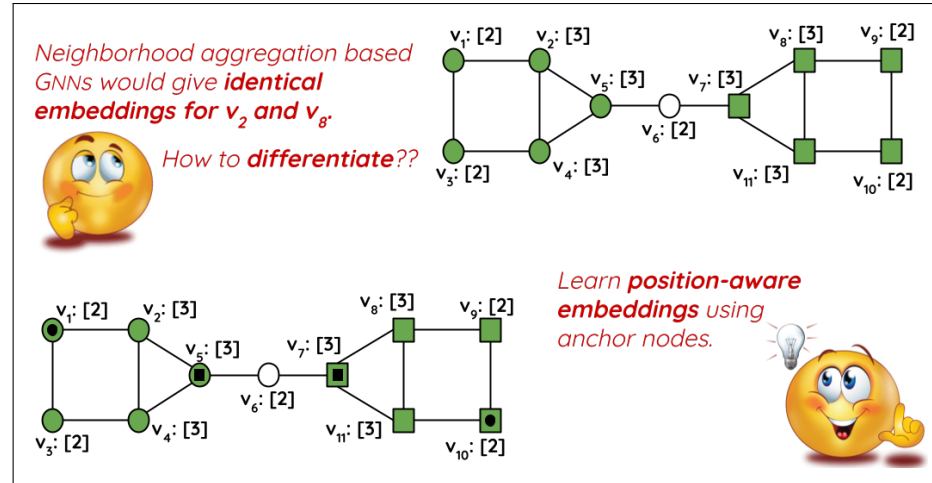


Figure: Traditional vs. position-aware GNNs.

- Most GNNs fail to distinguish nodes with similar neighbourhoods
- Traditional GNNs rely on neighborhood information or are *transductive*
- May be better in exploiting neighborhood feature distribution
- P-GNN [1] encapsulates position/ location of a node using shortest paths

## Contributions

**GRAPHREACH**: A node embedding learning model based on

- **Reachability estimations** between anchors and nodes, to aid incorporation of all paths, and
- **Strategic anchor selection** to cover all nodes and maximize differentiability of output embeddings, and is, thus,
- **Inductive**, i.e., can cater to previously unseen nodes and edges, and
- **Resilient** to adversarial attacks.

## GRAPHREACH

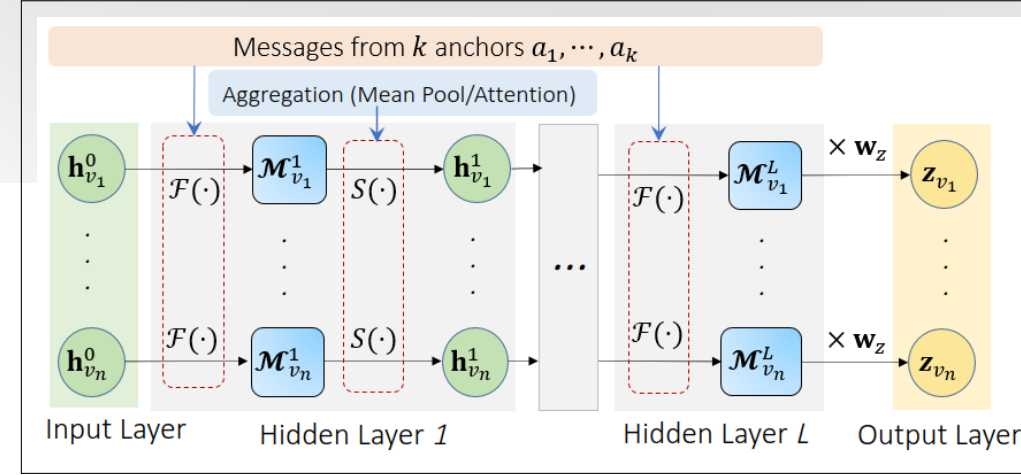


Figure: GRAPHREACH architecture

## Anchor Selection and Reachability Estimation

- Conduct multiple **random walks** from each node  $v$ .
- Form **bipartite graphs** from these random walks.
- Compute **marginal reachability** of vertices.
- Select  $k$  vertices with highest scores as **anchors**  $\mathcal{A} = (a_1, \dots, a_k)$ .
- **Reachability estimation**  $s(v, a_i)$  is *average* #times node  $v$  is visited across all walks from anchor  $a_i$ .

## Message Computation and Aggregation

- **Message Computation function**: Combine node attributes ( $h_v^l$ ) and position information.

$$\mathcal{F}(v, a, h_v^l, h_a^l) = \left( (s(v, a) \times h_v^l) \parallel (s(a, v) \times h_a^l) \right) \quad (1)$$

– Reachability estimations used in both directions to address asymmetry.

- **Message Matrix**  $\mathcal{M}_v^l$  for node  $v$  comprises linearly transformed messages from *all* anchors.

$$\mathcal{M}_v^l = \left( \bigoplus_{a \in \mathcal{A}} \mathcal{F}(v, a, h_v^l, h_a^l) \right) \cdot \mathbf{W}_{\mathcal{M}}^l \quad (2)$$

- **Aggregator**: *Mean-pooling*.

$$\mathcal{S}^M(\mathcal{M}_v^l) = \frac{1}{k} \sum_{i=1}^k \mathcal{M}_v^l[i] \quad (3)$$

**Output Layer**: Transform  $\mathcal{M}_v^L$  to output  $k$ -dimensional node embeddings.

$$\mathbf{z}_v \leftarrow \sigma(\mathcal{M}_v^L \cdot \mathbf{W}_Z) \quad \forall v \quad (4)$$

## Experimental Results

- GRAPHREACH is **better** than other architectures in real-world datasets

| Models     | Email                | Protein              | Models     | Communities          | PPI                  |
|------------|----------------------|----------------------|------------|----------------------|----------------------|
| GNN*       | 0.545 ± 0.012        | 0.528 ± 0.011        | GNN*       | 0.692 ± 0.049        | 0.803 ± 0.005        |
| P-GNN      | 0.640 ± 0.029        | 0.631 ± 0.175        | P-GNN      | 0.985 ± 0.008        | 0.808 ± 0.003        |
| GRAPHREACH | <b>0.949 ± 0.009</b> | <b>0.904 ± 0.003</b> | GRAPHREACH | <b>0.991 ± 0.003</b> | <b>0.810 ± 0.002</b> |

(a) Pairwise Node Classification (PNC)

(b) Link Prediction (LP)

Table: ROC AUC (GNN\*: Best accuracy obtained among GCN [2], GRAPHsAGE [3], GIN [4] and GAT [5])

- GRAPHREACH is more robust against graph modifications (**adversarial attacks**)

| Task | P-GNN |      |       | GRAPHREACH |      |              | Bf: ROC AUC before collusion<br>Af: ROC AUC after collusion<br>Δ: Change in accuracy due to collusion |
|------|-------|------|-------|------------|------|--------------|-------------------------------------------------------------------------------------------------------|
|      | Bf    | Af   | Δ     | Bf         | Af   | Δ            |                                                                                                       |
| PNC  | 0.92  | 0.82 | −0.10 | 1.00       | 0.98 | <b>−0.02</b> |                                                                                                       |
| LP   | 1.00  | 0.89 | −0.11 | 1.00       | 1.00 | <b>−0.00</b> |                                                                                                       |

Table: Robustness to adversarial attacks. (Dataset: Communities)

- GRAPHREACH utilizes **structural information** the best in absence of node features

| Task | Dataset  | GNN*        |      | P-GNN |      | GRAPHREACH |             | NC: Node Classification (Task)<br>S: Input is only the graph structure<br>S+T: Input consists of the graph structure with node attributes |
|------|----------|-------------|------|-------|------|------------|-------------|-------------------------------------------------------------------------------------------------------------------------------------------|
|      |          | S+T         | S    | S+T   | S    | S+T        | S           |                                                                                                                                           |
| NC   | CoRA     | <b>0.92</b> | 0.52 | 0.73  | 0.50 | 0.84       | <b>0.86</b> |                                                                                                                                           |
| NC   | CiteSeer | <b>0.82</b> | 0.52 | 0.73  | 0.55 | 0.75       | <b>0.71</b> |                                                                                                                                           |

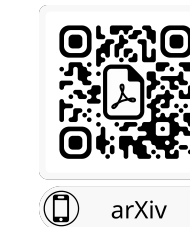
Table: ROC AUC (Traditional GNNs vs Position-aware GNNs).

- **Ablation**: Mean pooling is simpler but almost as good as attention aggregators.
- **Parameters**: Small number of random walks of short/medium lengths were enough.

## References

[1] P-GNN: ICML 2019; [2] GCN: ICLR, 2017; [3] GRAPHsAGE: NIPS, 2017; [4] GIN: ICLR, 2019; [5] GAT: ICLR, 2018.

## For more details



arXiv

Scan the QR codes.

← arXiv paper

github code →

If you have any questions, you can reach out to us!

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